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THE NEIGHBORHOOD TOPOLOGY CONVERTED FROM THE UNDIRECTED GRAPHS

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Abstract. In this paper, converting the topological graph to a special topology by adjacent vertices are studied. This topology is proved as a discrete topology. Where new definition of subbase denoted by NS_{G_τ} is introduced containing all sets of the vertices neighborhoods. The base NB_{G_τ} is obtained from the intersection of all elements of NS_{G_τ}. Then, the neighborhood topology $N\tau_{G_\tau}$ is generated by union of all elements of NB_{G_τ} with several examples.

Keywords: Topological graph, discrete topology, neighborhood topology.

AMS Subject Classification: 05C69.

1. Introduction.

The graph G is denoted by $G = (V, E)$, where $V(G)$ is a set of all vertices and $E(G)$ is a set of all edges in G. The vertex t adjacent with a vertex w if there is an edge between t and w. The number of all elements in $V(G)$ is called the order of G, denoted by $|V(G)|$. The open neighborhood of a vertex u defined as: $N(u) = \{t : t\,u \in E(G)\}$. For more information about graph theory see [1-17, 19, 26-28]. The discrete topology is denoted by (X, τ), where X is a non-empty set and τ is a family of all subsets of X, where $\tau = P(X)$. The set X and $\emptyset$ are belong to τ and both are open sets. The set $\mathcal{B} \subseteq \tau$ is called a base for τ if every open set in τ is a union of members of $\mathcal{B}$ [25]. A set $\sigma \subseteq \tau$ is called a subbase of τ if every open set in $\mathcal{B}$ is the finite intersection for elements of σ. Let $\{M_i;\ i \in I\}$ be a family of subset of X where if $I = \emptyset$, then $\bigcup_{i\in I} M_i = \emptyset$ and $\bigcap_{i\in I} M_i = X$ [29]. There are many papers joined graph theory and topology, see [18, 20-24]. In this paper, converting the topological graph to a discrete topology by adjacent vertices are studied. Where new definition of subbase is introduced containing all sets of the vertices neighborhoods. The base is obtained from the intersection of all elements of the subbase. Then, the neighborhood (discrete) topology is generated from the base with several examples.

2. Properties of Topological Graph.

In this section, we recall a special definition to construct a topological graph with some of its properties that proved in [24].

Definition 2.1 [24]: Let X be a non-empty set and τ be a discrete topology on X. The discrete topological graph denoted by $G_\tau = (V, E)$ is a graph of the vertex set $V = \{A; A \in \tau \text{ and } A \neq \emptyset, X\}$.
And the edge set $E = \{A\,B; A \nsubseteq B \text{ and } B \nsubseteq A\}$.
Proposition 2.2 [24]: Let X be a non-empty set of order n and let τ be a discrete topology on X. If $n = 2$, then $G_\tau \cong K_2$.
Proposition 2.3 [24]: Let X be a non-empty set of order n and let τ be a discrete topology on X. If $n = 3$, then $G_\tau \cong \overline{C_6}$.
Example 2.4 [24]: Let $|X| = 4$, then

$$\tau = \begin{cases}\emptyset, X, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\} \\ \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\end{cases},$$

so

$$V = \begin{Bmatrix}\{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \\ \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\end{Bmatrix}$$

Example 2.5 [24]: Let $|X| = 5$, then $\tau =$

$$\begin{Bmatrix}\emptyset, X, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \\ \{3,5\}, \{4,5\}, \{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,3,4\}, \{1,3,5\}, \{1,4,5\}, \{2,3,4\}, \{2,3,5\}, \\ \{2,4,5\}, \{3,4,5\}, \{1,2,3,4\}, \{1,2,3,5\}, \{1,2,4,5\}, \{1,3,4,5\}, \{2,3,4,5\}\end{Bmatrix},$$

so $V =$

$$\begin{Bmatrix}\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \\ \{3,5\}, \{4,5\}, \{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,3,4\}, \{1,3,5\}, \{1,4,5\}, \{2,3,4\}, \\ \{2,3,5\}, \{2,4,5\}, \{3,4,5\}, \{1,2,3,4\}, \{1,2,3,5\}, \{1,2,4,5\}, \{1,3,4,5\}, \{2,3,4,5\}\end{Bmatrix}.$$

Proposition 2.6 [24]: Let $|X| = n$, then the order of discrete topological graph G_τ is $2^n - 2$.

3. Topological Space Generated by Topological Graphs

In this section, converting the topological graph to a discrete topology are studied.
Definition 3.1 [19]: Let G_τ be a topological graph where the neighborhood of a vertex u_i for any $u_i \in V(G_\tau)$ defined as:

$$N(u_i) = \{u_j \in V(G_\tau): u_j \text{ adjacent with } u_i \text{ where } i \neq j\}$$

Definition 3.2: Let NS_{G_τ} be a collection of subsets of neighborhoods of V whose union equals V. Such that $NS_{G_\tau}(V) = \{N(u_i)\}_{u_i \in V(G_\tau)}$ for all $u_i \in V(G_\tau), i = 1,2, \dots, 2^n - 2$, and NS_{G_τ} forms a subbase. The topology that generated by NS_{G_τ} is defined to be collection of all unions of finite intersection of elements of NS_{G_τ}.

Definition 3.3: Let NB_{G_τ} be a basis generated by finite intersection of members of $NS_{G_\tau}(V)$. Where it defined as follow:

$$NB_{G_\tau}(V) = \{A; A \subseteq V, A \text{ is a finite intersection of members of } NS_{G_\tau}\}$$

Definition 3.4: Let $N\tau_{G_\tau}$ be a topology on a set V generated by NB_{G_τ} and NS_{G_τ} called neighborhood topology of a graph G_τ.

Example 3.5: Let G_τ be a topological graph for $|X| = 2$. We find the neighborhood topology $N\tau_{G_\tau}$ of G_τ.

Let $V = \{u_1, u_2\}$, where: $u_1 = \{1\}$ and $u_2 = \{2\}$. Then,

$N(u_1) = \{u_2\}$, and $N(u_2) = \{u_1\}$.

Then, $NS_{G_\tau}(V) = \{\{u_1\},\{u_2\}\}$ and $NB_{G_\tau}(V) = \{\emptyset, V, \{u_1\}, \{u_2\}\}$.

$N\tau_{G_\tau}(V) = \{\emptyset, V, \{u_1\}, \{u_2\}\}$ is discrete topology. See Figure 1.

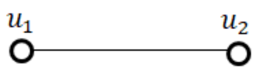

Figure 1: The topological graph for $|X| = 2$.

Example 3.6: Let G_τ be a topological graph for $|X| = 3$. Thus, we extract the neighborhood topology $N\tau_{G_\tau}$ of topological graph G_τ.

Let

$V = \{u_1, u_2, u_3, u_4, u_5, u_6\}$.

Where $u_1 = \{1\}, u_2 = \{2\}, u_3 = \{3\}, u_4 = \{1,2\}, u_5 = \{1,3\}, u_6 = \{2,3\}$.

Then,

$N(u_1) = \{u_2, u_3, u_6\}, N(u_2) = \{u_1, u_3, u_5\}, N(u_3) = \{u_1, u_2, u_4\}$,

$N(u_4) = \{u_3, u_5, u_6\}, N(u_5) = \{u_2, u_4, u_6\}, N(u_6) = \{u_1, u_4, u_5\}$,

$NS_{G_\tau}(V) = \{\{u_2, u_3, u_6\}, \{u_1, u_3, u_5\}, \{u_1, u_2, u_4\}, \{u_3, u_5, u_6\}, \{u_2, u_4, u_6\}, \{u_1, u_4, u_5\}\}$.

By taking the intersection of sets of $NS_{G_\tau}(V)$ we get the base as:

$$NB_{G_\tau} = \{\emptyset, V, \{u_1\}, \{u_2\}, \{u_3\}, \{u_4\}, \{u_5\}, \{u_6\}, \{u_1, u_4\}, \{u_1, u_5\}, \{u_2, u_4\}, \{u_2, u_6\}, \{u_3, u_5\}, \{u_3, u_6\}, \{u_1, u_2, u_4\}, \{u_1, u_3, u_5\}, \{u_1, u_4, u_5\}, \{u_2, u_3, u_6\}, \{u_2, u_4, u_6\}, \{u_3, u_5, u_6\}\}.$$

By taking all unions. The neighborhood topology can be written as:

u_1 u_2

$N\tau_{G_\tau}(V)$
$= \{\emptyset, V, \{u_1\}, \{u_2\}, \{u_3\}, \{u_4\}, \{u_5\}, \{u_6\}, \{u_1,u_2\}, \{u_1,u_3\},$
$\{u_1,u_4\}, \{u_1,u_5\}, \{u_1,u_6\}, \{u_2,u_3\}, \{u_2,u_4\}, \{u_2,u_5\}, \{u_2,u_6\}, \{u_3,u_4\},$
$\{u_3,u_5\}, \{u_3,u_6\}, \{u_4,u_5\}, \{u_4,u_6\}, \{u_5,u_6\}, \{u_1,u_2,u_3\}, \{u_1,u_2,u_4\},$
$\{u_1,u_2,u_5\}, \{u_1,u_2,u_6\}, \{u_1,u_3,u_4\}, \{u_1,u_3,u_5\}, \{u_1,u_3,u_6\}, \{u_1,u_4,u_5\},$
$\{u_1,u_4,u_6\}, \{u_1,u_5,u_6\}, \{u_2,u_3,u_4\}, \{u_2,u_3,u_5\}, \{u_2,u_3,u_6\}, \{u_2,u_4,u_5\},$
$\{u_2,u_4,u_6\}, \{u_2,u_5,u_6\}, \{u_3,u_4,u_5\}, \{u_3,u_4,u_6\}, \{u_3,u_5,u_6\}, \{u_4,u_5,u_6\},$
$\{u_1,u_2,u_3,u_4\}, \{u_1,u_2,u_3,u_5\}, \{u_1,u_2,u_3,u_6\}, \{u_1,u_2,u_4,u_5\},$
$\{u_1,u_2,u_4,u_6\}, \{u_1,u_2,u_5,u_6\}, \{u_1,u_3,u_4,u_5\}, \{u_1,u_3,u_4,u_6\},$
$\{u_1,u_3,u_5,u_6\}, \{u_1,u_4,u_5,u_6\}, \{u_2,u_3,u_4,u_5\}, \{u_2,u_3,u_4,u_6\},$
$\{u_2,u_3,u_5,u_6\}, \{u_2,u_4,u_5,u_6\}, \{u_3,u_4,u_5,u_6\}, \{u_1,u_2,u_3,u_4,u_5\},$
$\{u_1,u_2,u_3,u_4,u_6\}, \{u_1,u_2,u_3,u_5,u_6\}, \{u_1,u_2,u_4,u_5,u_6\},$
$\{u_1,u_3,u_4,u_5,u_6\}, \{u_2,u_3,u_4,u_5,u_6\}\}.$
Then, $N\tau_{G_\tau}$ is discrete topology. See Figure 2.

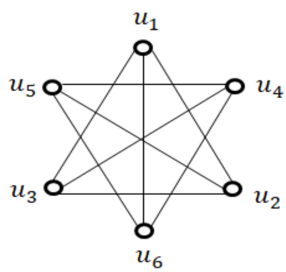

Figure 2: The topological graph for $|X| = 3$.

Example 3.7: Let G_τ be a topological graph for $|X| = 4$. We find the neighborhood topology $N\tau_{G_\tau}$ of G_τ.

Let $V = \{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8,u_9,u_{10},u_{11},u_{12},u_{13},u_{14}\}$. Where $u_1 = \{1\}, u_2 = \{2\}, u_3 = \{3\}, u_4 = \{4\}, u_5 = \{1,2\}, u_6 = \{1,3\}, u_7 = \{1,4\}, u_8 = \{2,3\}, u_9 = \{2,4\}, u_{10} = \{3,4\}, u_{11} = \{1,2,3\}, u_{12} = \{1,2,4\}, u_{13} = \{1,3,4\}, u_{14} = \{2,3,4\}$. Then,

$N(u_1) = \{u_2,u_3,u_4,u_8,u_9,u_{10},u_{14}\}, N(u_2) = \{u_1,u_3,u_4,u_6,u_7,u_{10},u_{13}\},$
$N(u_3) = \{u_1,u_2,u_4,u_5,u_7,u_9,u_{12}\}, N(u_4) = \{u_1,u_2,u_3,u_5,u_6,u_8,u_{11}\},$
$N(u_5) = \{u_3,u_4,u_6,u_7,u_8,u_9,u_{10},u_{13},u_{14}\},$
$N(u_6) = \{u_2,u_4,u_5,u_7,u_8,u_9,u_{10},u_{12},u_{14}\}.$
In the similar way above we find $N(u_i), i = 7,8,\ldots,14$.

Where $NS_{G_\tau}(V) = \{N(u_i)\}_{u_i \in V(G_\tau)}$, for all $i = 1,2,3,\dots,14$.
And by taking the intersection of sets of $NS_{G_\tau}(V)$ we get the base as:
NB_{G_τ}
$= \{\emptyset, V, \{u_1\}, \{u_2\}, \{u_3\}, \{u_4\}, \{u_5\}, \{u_6\}, \{u_7\}, \{u_8\}, \{u_9\}, \{u_{10}\}, \{u_{11}\},$
$\{u_{12}\}, \{u_{13}\}, \{u_{14}\}, \{u_1, u_5\}, \{u_1, u_6\}, \{u_1, u_7\}, \{u_2, u_5\}, \{u_2, u_8\}, \{u_2, u_9\},$
$\{u_3, u_6\}, \{u_3, u_8\}, \{u_3, u_{10}\}, \{u_4, u_7\}, \{u_4, u_9\}, \{u_4, u_{10}\}, \{u_5, u_6\}, \{u_5, u_7\},$
$\{u_5, u_8\}, \{u_5, u_9\}, \{u_5, u_{10}\}, \{u_5, u_{11}\}, \{u_5, u_{12}\}, \{u_6, u_7\}, \{u_6, u_8\}, \{u_6, u_9\},$
$\{u_6, u_{10}\}, \{u_6, u_{11}\}, \{u_6, u_{13}\}, \{u_7, u_8\}, \{u_7, u_9\}, \{u_7, u_{10}\}, \{u_7, u_{12}\},$
$\{u_7, u_{13}\}, \{u_8, u_9\}, \{u_8, u_{10}\}, \{u_8, u_{11}\}, \{u_8, u_{14}\}, \{u_9, u_{10}\}, \{u_9, u_{12}\},$
$\{u_9, u_{14}\}, \{u_{10}, u_{13}\}, \{u_{10}, u_{14}\}, \{u_1, u_2, u_5\}, \{u_1, u_3, u_6\}, \{u_1, u_4, u_7\},$
$\{u_1, u_5, u_6\}, \{u_1, u_5, u_7\}, \{u_1, u_6, u_7\}, \{u_2, u_3, u_8\}, \{u_2, u_4, u_9\}, \{u_2, u_5, u_8\},$
$\{u_2, u_5, u_9\}, \{u_2, u_8, u_9\}, \{u_3, u_4, u_{10}\}, \{u_3, u_6, u_8\}, \{u_3, u_6, u_{10}\},$
$\{u_3, u_8, u_{10}\}, \{u_4, u_7, u_9\}, \{u_4, u_7, u_{10}\}, \{u_4, u_9, u_{10}\}, \{u_5, u_6, u_9\},$
$\{u_5, u_6, u_{10}\}, \{u_5, u_6, u_{11}\}, \{u_5, u_7, u_8\}, \{u_5, u_7, u_{10}\}, \{u_5, u_7, u_{12}\},$
$\{u_5, u_8, u_{10}\}, \{u_5, u_8, u_{11}\}, \{u_5, u_9, u_{10}\}, \{u_5, u_9, u_{12}\}, \{u_5, u_{11}, u_{12}\},$
$\{u_6, u_7, u_8\}, \{u_6, u_7, u_9\}, \{u_6, u_7, u_{10}\}, \{u_6, u_7, u_{13}\}, \{u_6, u_8, u_9\},$
$\{u_6, u_8, u_{11}\}, \{u_6, u_9, u_{10}\}, \{u_6, u_{10}, u_{13}\}, \{u_6, u_{11}, u_{13}\}, \{u_7, u_8, u_9\},$
$\{u_7, u_8, u_{10}\}, \{u_7, u_9, u_{12}\}, \{u_7, u_{10}, u_{13}\}, \{u_7, u_{12}, u_{13}\}, \{u_8, u_9, u_{14}\}, \dots\dots\dots\}.$
By similar technique of Example 3.6 we find the neighborhood topology $N\tau_{G_\tau}(V)$.
Such that the number of all sets of $N\tau_{G_\tau}$ is 2^{14}. Thus, $N\tau_{G_\tau}$ is discrete topology. See Figure 3.

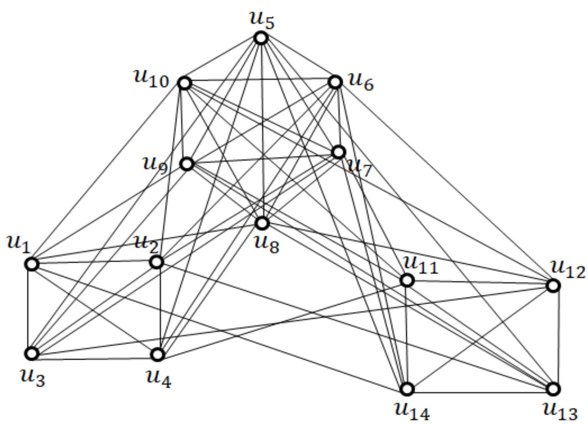

Figure 3: The topological graph for $|X| = 4$.

Example 3.8: Let $|X| = 5$ and G_τ be a topological graph. To find the neighborhood topology $N\tau_{G_\tau}$ of topological graph G_τ.
Let

$V=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8,u_9,u_{10},u_{11},u_{12},u_{13},u_{14},u_{15},u_{16},u_{17},$
$u_{18},u_{19},u_{20},u_{21},u_{22},u_{23},u_{24},u_{25},u_{26},u_{27},u_{28},u_{29},u_{30}\}$.
Where
$u_1=\{1\},u_2=\{2\},u_3=\{3\},u_4=\{4\},u_5=\{5\},u_6=\{1,2\},u_7=\{1,3\},\quad u_8=\{1,4\},\quad u_9=\{1,5\},\quad u_{10}=\{2,3\},\quad u_{11}=\{2,4\},\ u_{12}=\{2,5\},\quad u_{13}=\{3,4\},\quad u_{14}=\{3,5\},\quad u_{15}=\{4,5\},$
$u_{16}=\{1,2,3\},\ u_{17}=\{1,2,4\},\quad u_{18}=\{1,2,5\},\quad u_{19}=\{1,3,4\},\quad u_{20}=\{1,3,5\},\ u_{21}=\{1,4,5\},u_{22}=\{2,3,4\},u_{23}=\{2,3,5\},u_{24}=\{2,4,5\},u_{25}=\{3,4,5\},u_{26}=\{1,2,3,4\},u_{27}=\{1,2,3,5\},u_{28}=\{1,2,4,5\},u_{29}=\{1,3,4,5\},u_{30}=\{2,3,4,5\}$
Then,
$N(u_1)=\{u_2,u_3,u_4,u_5,u_{10},u_{11},u_{12},u_{13},u_{14},u_{15},u_{22},u_{23},u_{24},u_{25},u_{30}\}$,
$N(u_2)=\{u_1,u_3,u_4,u_5,u_7,u_8,u_9,u_{13},u_{14},u_{15},u_{19},u_{20},u_{21},u_{25},u_{29}\}$,
$N(u_3)=\{u_1,u_2,u_4,u_5,u_6,u_8,u_9,u_{11},u_{12},u_{15},u_{17},u_{18},u_{21},u_{24},u_{28}\}$,
$N(u_4)=\{u_1,u_2,u_3,u_5,u_6,u_7,u_9,u_{10},u_{12},u_{14},u_{16},u_{18},u_{20},u_{23},u_{27}\}$,
$N(u_5)=\{u_1,u_2,u_3,u_4,u_6,u_7,u_8,u_{10},u_{11},u_{13},u_{16},u_{17},u_{19},u_{22},u_{26}\}$,
$N(u_6)=\{u_3,u_4,u_5,u_7,u_8,u_9,u_{10},u_{11},u_{12},u_{13},u_{14},u_{15},u_{19},u_{20},u_{21},u_{22},$
$u_{23},u_{24},u_{25},u_{29},u_{30}\}$.
In the similar way above we find $N(u_i),i=7,8,\dots,30$.
Such that $NS_{G_\tau}(V)=\{N(u_i)\}_{u_i\in V(G_\tau)}$, for all $i=1,2,3,\dots,30$. We find NB_{G_τ} and $N\tau_{G_\tau}$ by the same technique of Example 3.6. So, we get NB_{G_τ} which has all sets of singleton u_i where $\{u_i\}\in NB_{G_\tau}$, for all $i=1,2,3,\dots,30$, and $V\in NB_{G_\tau}$. Since $N\tau_{G_\tau}$ is the union of all sets of NB_{G_τ}. Then, the number of all sets of $N\tau_{G_\tau}$ is 2^{30} and it is discrete topology. See Figure 4. Also, if $n>5$ the topological space generated by topological graph is discrete topology.

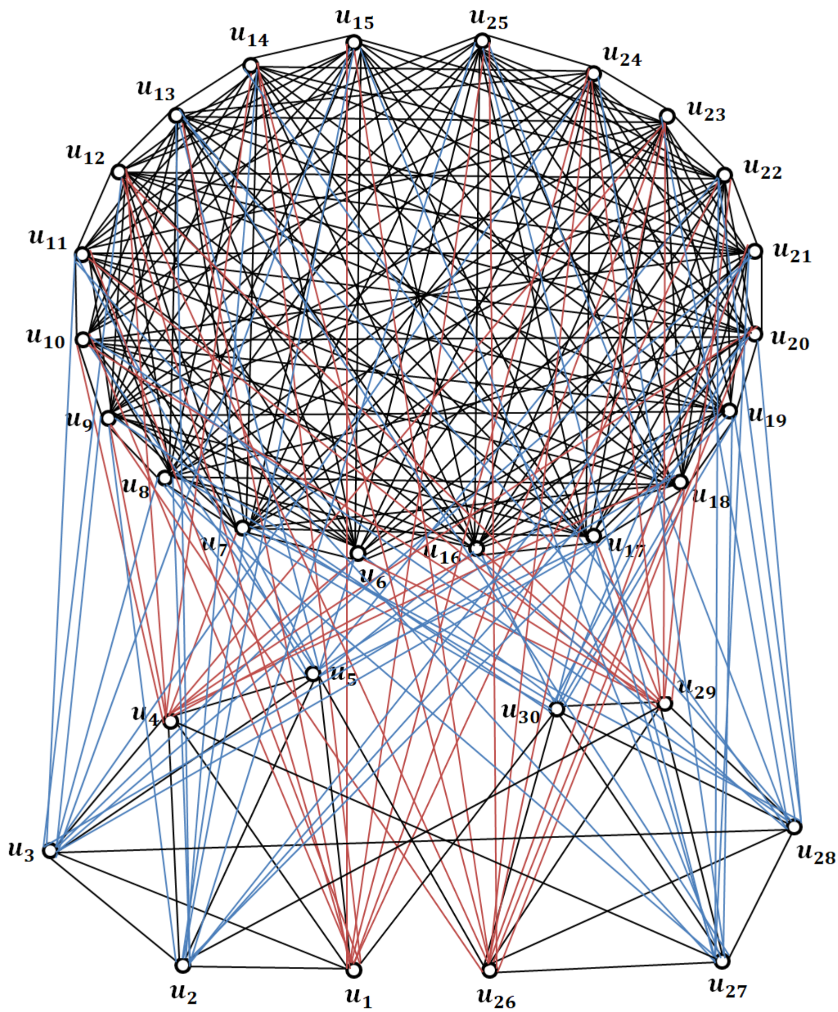

Figure 4: The topological graph for $|X| = 5$.

4. Open problems

1- Converting the topological graph to the discrete topology by other ways, by the adjacent or non-adjacent edges or vertices.

2- Apply many types of domination parameters on the topological graph such as: Pitchfork domination, arrow domination, co-independent domination.

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